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ROBOTICS

Automatic Grasping of Objects by a Manipulator Equipped with a Multifinger Hand

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Abstract—We consider the problem of controlling a manipulator during the operation of the automatic grasping of a noncooperative object. The manipulator is equipped with a gripper in the form of a multifinger hand. The solution of the problem includes the planning and execution stages. When planning, the coordinates of contact points on the surface of the object, along with the coordinates of the manipulator and the fingers of the hand at the instant of a grasping are determined. When executing, the manipulator and the hand move from the initial position into the planned position.

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INTRODUCTION

The term *grasping* defines a process whose result is a mechanical connection of a manipulator and an object. At present, much attention is placed on grasping in the automatic mode [1, 2], i.e., without a human intervention. We know two main methods of automatic grasping of objects:

(a) using special fastenings placed on the surface of a grasped object, along with docking devices located on the flange of the manipulator;

(b) without the use of fastenings on the object and docking devices on the manipulator.

Objects equipped with special fastenings are called cooperative. Cooperative objects grasping consists of joining the docking devices on the manipulator flange to fastenings on an object. This method has been successfully implemented in practice for a long time, e.g., in operating the SSRMS, SPDM, and JEMRMS manipulators of the International Space Station [3].

However, most objects do not have special fastenings: they are not cooperative. Examples include details in robotic industrial assembly or sorting of subjects [4], objects of service robots [5], and (for space robots) freely moving objects (satellites, asteroids, and space debris [6]). In order to grasp such objects, manipulators are equipped with grippers in the form of hands with several fingers.

A noncooperative object is grasped by wrapping the robotic hand fingers. The grasping operation consists of two parts: planning and executing. In planning, the coordinates of points for which the object will be grasped (contact points), the coordinates of the joints of the hand fingers, and the forces applied to the object at the contact points and moments relative to these points are determined [2]. In executing, the manipulator moves the hand from the initial position into the position where the hand fingers come in contact with the object at the planned points. After that, the object is compressed with the fingers.

This paper assumes that both parts of the grasping operation (planning and executing) are performed in the automatic mode. The operation of moving the hand fingers and the manipulator itself is analogous to the operation of moving the manipulator into a given position. The algorithm of implementing this operation has received sufficient coverage in the literature (see, e.g., [7]) and is not considered here.

1. GRASP PLANNING

Problems of the theory of robotic grasping and practical aspects of its planning have aroused the interest of specialists in different countries, as illustrated by the numerous publications devoted to this subject [1, 2, 8–10]. Computer systems of planning and simulation of object grasping by grippers of various types have been widely accepted. Examples are found in GraspIt! (a simulating system created in Robotics Lab of Columbia University) [11] and the OpenRAVE (open robotics automation virtual environment) package [12] developed at the Robotics Institute of Carnegie–Mellon University. These systems are open; they are actively used in practical applications, specifically, in the robotic setup developed at Bauman State Technical University [13].

In modern manipulation systems, grippers often have the form of a multifinger hand [8, 9]. For a hand, grasp is planned. Here, a manipulator (an arm) is considered only as a means of moving the hand to an object when performing an operation. In what follows for brevity we denote such a system by hand + object (HO).

This paper proposes a different approach, in which grasp is planned for an entire manipulation system, including both the hand and the arm (in what follows, we denote such a system by arm + hand + object (AHO)). The AHO system has a larger number of degrees of freedom compared to the HO system. This makes it possible, first, to form more variants of grasping and, second, to find a solution in cases where suitable grasping in the HO system cannot be found. Such an approach to grasp planning is proposed for the first time.

2. BASIC RELATIONS OF ROBOTIC GRASPING THEORY

The robotic grasping theory considers relations [2], which connect the following vectors:

(a) the vector ψ of external forces (applied to the center of mass of an object) and moments about the center of mass, and the block vector λ formed of vectors λ_i, $i = 1, 2, \ldots, m$ (λ_i includes, in general, vectors of force applied to the object by the hand at the contact point of the ith finger and the object (the ith grasp point), and the moment about this point; the moment vector is also considered to be applied to the object from the ith finger (m is the number of contact points); and m is equal to the number of fingers of the hand);

(b) the block vector v, components of which are the vectors of linear and angular speeds of the center mass of the object, and the vector v_s formed of m vectors v_{si}, which in the general case contain linear-speed vectors of the ith contact point and projections of the vector of the angular speed of the object rotation relative to the center of mass on the normal to the surface of the object at contact points (below the vectors λ_i and v_{si} are considered in detail);

(c) the vector μ of forces (or moments, depending on whether the appropriate actuators performs translational or rotational movement of adjacent links) of finger joints actuators, and the vector λ;

(d) the vector $\dot{q}$ of time derivatives of the coordinates of the finger joints, and the block vector v_s.

The dimension of the vectors ψ and v is n_v. Here, n_v depends on whether we consider the object movement of in a plane or in space. In the first case, $n_v = 3$; in the second case, $n_v = 6$. The dimension of the vectors μ and $\dot{q}$ is equal to the number of joints of the hand.

These relations have the following form:

$$G\lambda = -\psi, \tag{2.1}$$

$$G^{\mathrm{T}}v = v_s, \tag{2.2}$$

$$J^{\mathrm{T}}\lambda = \mu, \tag{2.3}$$

$$J\dot{q} = v_s. \tag{2.4}$$

In expressions (2.1)–(2.4), the matrix G is a grasp matrix and J is the hand Jacobian. Vectors that represent components of the vectors ψ and v are assigned in the inertial frame related to the base of the manipulator, while components of the vectors λ and v_s are assigned in frames, whose origins are located at the grasping points. The x_i-axis of these frames is normal to the tangent plane at a contact point in the direction to an object. The other two axes are located in the tangent plane and form the right-handed frame of reference. Under the influence of external forces and moments, along with forces applied to contact points and vectors of moments about these points (applied by the hand), the object is in equilibrium. Because the movements of the hand's fingers and of the object during grasping are small, the relations for the speed vectors v, v_s, and $\dot{q}$ remain valid also for motion vectors. In formulas (2.2) and (2.3), superscript T for the matrices G and J means the transposition.

Relations (2.1)–(2.4) are considered for three types of contacts of fingers and an object [2, 14]:

(a) the point contact of an object (an absolutely solid body) with a hand whose fingers are absolutely solid bodies; here, forces of friction at the contact points of the object and the hand are not taken into account;

(b) the contact point of an object (an absolutely solid body) with a hand whose fingers are absolutely solid bodies; here, forces of friction are taken into account;

(c) the contact of an object (an absolutely solid body) with a hand whose fingers can be deformed in the direction normal to the point of contact (soft fingers); here, it is assumed that the surface friction and the contact patch at the ith point are sufficiently large.

Depending on the type of contact, the dimensions of the components of the vectors λ and v_s (i.e., λ_i and v_{si}) included in (2.1)–(2.4) are changed. In a frictionless point-contact of the ith finger, the object experiences a force directed normal to its surface; here, the contact point moves only forward in the direction normal to this surface. The vectors λ_i and v_{si} each contain one component equal to the force and the displacement in the direction of the normal (in the direction to the object), respectively.

If the ith finger comes into contact with the object at one point, with the friction forces taken into account, then the vector of the contact force can deviate from the normal. In this case, λ_i contains three components, which represent projections of the vector forces on the axes of the frame (associated with the object) with the origin at the ith point; v_{si} also contains three components: the projections of the linear-speed vector of the ith point on the axes of the contact frame.

If the ith finger is “soft”, then v_{si} contains four components: the projections of the linear-speed vector of the ith contact point of the corresponding finger on the axes of the contact frame, along with one component of the angular speed of rotation about the normal. Here, λ_i also contains four components.

Consider the form of the matrices G and J. The grasp matrix G has the form

$$G = \begin{bmatrix} G_{11} & G_{12} & \dots & G_{1m} \\ G_{21} & G_{22} & \dots & G_{2m} \\ \dots & \dots & \dots & \dots \\ G_{n_v 1} & G_{n_v 2} & \dots & G_{n_v m} \end{bmatrix}. \quad (2.5)$$

The number of rows of the matrix G is n_v, i.e., the number of degrees of freedom of an object. The number of columns depends on the number m of contact points and the type of contact at each point. The element G_{ij} represents a row-vector whose dimension lj is equal to the dimension of the vector λ_j. In the case of a frictionless point-contact at the jth point, the element G_{ij} of the matrix G is a 1-by-1 element. If friction is found, then G_{ij} is a 1-by-3 element (the object and the hand’s fingers are absolutely solid bodies). In grasping an object by soft fingers, G_{ij} is a 1-by-4 element. Note that in the general case, the types of contacts at particular points can vary. We use designation $l = l_1 + \dots + l_m$. Then the matrix G is an n_v-by-l matrix.

The hand Jacobian is a diagonal matrix:

$$J = \operatorname{diag}(J_1, \dots J_i \dots J_m), \quad (2.6)$$

where J_i is the Jacobian of the ith finger; J_i relates the vector $\dot{q}_i$ of derivatives of coordnates for joints of the ith finger to the vector v_{si} of the linear speed of the ith contact point and the angular speed of the rotation of the object about the ith point.

The number of rows of the matrix J is equal to l and the number of columns is equal to the total number of joints of all fingers of the hand.

3. THE DESIRED GRASP PROPERTIES

In order to specify the grasp properties, it is necessary to find a solution of system of linear vector-matrix Eqs. (2.1)–(2.4). The solution of these equations implies the inversion of the matrices on the left side. In the general case, these matrices can have an incomplete rank. Consider the solution by the example of Eq. (2.1).

In accordance with [15], we present λ as

$$\lambda = \lambda_r + \lambda_0,$$

where the vector λ_r belongs to the rank space $R(G^{\mathrm{T}})$ of the matrix G^{T}, while the vector x_0 belongs to the null space $N(G)$ of the matrix G. The last expression can be written as

$$\lambda = -G^{+}\psi + N(G)\alpha, \tag{3.1}$$

where G^{+} is pseudoinverse of a matrix for G and α is an arbitrary vector-multiplier. Analogously, we obtain

$$\nu = (G^{\mathrm{T}})^{+}v_s + N(G^{\mathrm{T}})\beta, \tag{3.2}$$

$$\lambda = (J^{\mathrm{T}})^{+}\mu + N(J^{\mathrm{T}})\gamma, \tag{3.3}$$

$$\dot{q} = J^{+}v_s + N(J)\delta. \tag{3.4}$$

Eqs. (3.2)–(3.4), $N(\cdot)$ contain the null spaces of the corresponding matrices; β, γ, and δ are arbitrary vector-multipliers; and $(\cdot)^{+}$ are matrices pseudoinverse for $(\cdot)$.

Using the properties of the null spaces of grasp matrices and Jacobians, the grasp properties are described as follows [2].

1. A grasp is ***sufficient*** (in order to hold an object) if $N(G)$ is nontrivial. It follows from (3.1) that in this case, λ_i contains internal forces and moments, which affect only the intensity of the compression of the object by the hand's fingers. The fulfilment of the nontriviality condition for $N(G)$ is one of the desired properties of the grasp.

2. A grasp is termed ***uncertain*** if $N(G^{\mathrm{T}})$ is nontrivial. It follows from (3.2) that in this case, an object moves and this movement is not associated with the movement of the fingers at the contact points; this points to a lack of control of the object's movement by the hand's fingers and is unacceptable.

3. A grasp is termed ***defective*** if $N(J^{\mathrm{T}})$ is nontrivial. It follows from (3.3) that here we cannot guarantee the availability of a vector of forces (or moments) μ developed by the actuators of the fingers joints that provides the assigned vector of forces and moments λ (relation (3.1)) at the contact points. In other words, $N(J^{\mathrm{T}})$ contains vectors λ that do not depend on the forces (or moments) of actuators of joints μ.

4. A grasp is termed ***redundant*** if $N(J)$ is nontrivial. Here (it follows from Eq. (3.4)), the vector $\dot{q}$ contains components that are not associated with the object's movement.

In the grasping theory we have the key relations whose use makes it possible to execute the required object's movement and forces (moments), applied by the hand to the object, and to control these movements and applied forces (moments). These relations determine the desired properties of a grasp. The correspondence of the grasp with the desired properties can be set by analyzing Eqs. (3.1)–(3.4). Considering Eqs. (3.1)–(3.4), we see the following points.

1. By manipulating the hand's fingers, we can move the object (the speed vector v); here, we exclude the spontaneous movement if $N(G^{\mathrm{T}})$ is trivial. The formulated conditions are equivalent to the requirement $\dim N(G^{T}) = 0$ or $\operatorname{rank} G = n_v$. The latter condition is imposed on the grasp and in order to ensure the force effects (by the hand's fingers on the object) that compensate the action of vector ψ (3.1).

2. Controlling the movement of the object with the hand's fingers (the speed vector of the fingers' movement $\dot{q}$) requires the simultaneous fulfilment of the conditions $\dim N(G^{\mathrm{T}}) = 0$ and $\operatorname{rank} GJ = n_v$. These conditions are equivalent to the condition $\operatorname{rank} GJ = \operatorname{rank} G = n_v$, which follows from the joint consideration of relations (3.2) and (3.4). These properties must be fulfilled for ensuring the control of the force effects on the object.

3. From Eq. (3.1) it follows that if $N(G)$ is nontrivial, then there exist forces (internal) that compress an object. In this case, not all internal forces in $N(G)$ can be controllable. It is shown in [1, 2] that all internal forces in $N(G)$ are controllable if and only if there is no intersection of the null-spaces $N(G)$ and $N(J^{\mathrm{T}})$, i.e., $N(G) \cap N(J^{\mathrm{T}}) = 0$.

4. JACOBIANS OF THE HO AND AHO SYSTEMS

We present (2.6) for the hand Jacobian as follows:

$$J_{\mathrm{HO}} = \operatorname{diag}(J_{\mathrm{HO}1}, \ldots J_{\mathrm{HO}i} \ldots J_{\mathrm{HO}m}).$$

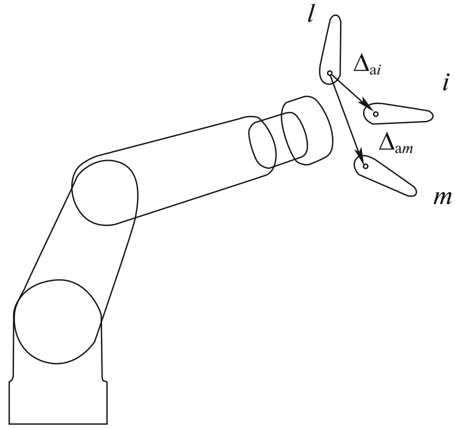

Fig. 1. Schematic model of hand with m fingers.

The HO index is added for the convenience of considering the Jacobians of the HO and AHO systems.

The Jacobian for the AHO system is presented as follows:

$$J_{\text{AHO}} = \begin{bmatrix} J_{\text{HO1}} & 0 & \dots & 0 & J_{\text{a1}} \\ 0 & J_{\text{HO2}} & \dots & 0 & J_{\text{a2}} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & J_{\text{HO}m} & J_{\text{a}m} \end{bmatrix} = [J_{\text{HO}} \ \ J_{\text{a}}]. \tag{4.1}$$

Here, $J_{\text{a}i}$ is a Jacobian, which links the time derivative of the vector of coordinates of joints of the manipulator's hand with a (block) vector of the linear and angular speeds of a point in the base of the ith finger; J_{a} is a block matrix composed of the matrices $J_{\text{a}i}$, $i = 1,2, \dots, m$.

It can be seen that the matrix J_{AHO} of the AHO system differs from the analogous matrix J_{HO} of the HO system by an additional column: J_{HO} is an l-by-n_{qs} matrix and J_{AHO} is an l-by-$(n_{qs} + n_{q\text{a}})$ matrix. Here, n_{qs} is the total number of joints of all fingers of the hand and $n_{q\text{a}}$ is the number of joints of the manipulator's arm. It is assumed that each joint allows a relative displacement of adjacent links along or around only one axis.

The components of $J_{\text{a}i}$ are presented as follows:

$$J_{\text{a}i} = J_{\text{a}1} + \Delta_{\text{a}i},$$

where $J_{\text{a}1}$ is the Jacobian of a manipulator's arm for the frame located in the base of the first finger and $\Delta_{\text{a}i}$ are correction matrices. The availability of $\Delta_{\text{a}i}$ is associated with various positions of the fingers' bases on the hand's palm relative to the base of the first finger (Fig. 1).

Then the following relation is true:

$$v_s = J_{\text{AHO}}[\dot{q}^{\text{T}} \ \ \dot{q}_{\text{a}}^{\text{T}}]^{\text{T}},$$

where $\dot{q}_{\text{a}}$ is a vector composed of derivatives of the coordinates of the arm joints; the dimension of this vector is the same as the number of links (joints) of the arm.

5. PROPERTIES OF RANK AND NULL SPACES OF JACOBIANS OF THE HO AND AHO SYSTEMS

As can be seen from (2.5), the grasp matrix G is the same for the HO and AHO systems. Comparing (2.6) and (4.1), we see that the Jacobians J_{HO} and J_{AHO} differ: the matrix J_{AHO} has an additional column of the block matrices $J_{\text{a}i}$. We estimate the influence of the properties of the matrix J_{a} on the ability to ensure the desired properties of a grasp, including the case where these properties cannot be ensured by considering the grasp of an object in the HO system. For the dimensions of the rank and null spaces of matrices, the following relations are true:

$$\dim R(J_{\text{HO}}) + \dim N(J_{\text{HO}}^{\text{T}}) = n_{J_{\text{HO}}},$$

$$\dim R(J_{\mathrm{HO}}^{\mathrm{T}}) + \dim N(J_{\mathrm{HO}}) = k_{J_{\mathrm{HO}}},$$

$$\dim R(J_{\mathrm{AHO}}) + \dim N(J_{\mathrm{AHO}}^{\mathrm{T}}) = n_{J_{\mathrm{AHO}}},$$

$$\dim R(J_{\mathrm{AHO}}^{\mathrm{T}}) + \dim N(J_{\mathrm{AHO}}) = k_{J_{\mathrm{AHO}}},$$

where $n_{J_{\mathrm{HO}}}$ is the number of rows of the matrix J_{HO}, $k_{J_{\mathrm{HO}}}$ is the number of columns of the matrix J_{HO}, $n_{J_{\mathrm{AHO}}}$ is the number of rows of the matrix J_{AHO}, and $k_{J_{\mathrm{AHO}}}$ is the number of columns of the matrix J_{AHO}.

The number of columns of the matrix J_{AHO} (4.1) of the AHO system is larger than the number of columns of the matrix J_{HO}; however, the numbers of rows of these matrices are identical; i.e., $n_{J_{\mathrm{AHO}}} = n_{J_{\mathrm{HO}}}$ and $k_{J_{\mathrm{AHO}}} > k_{J_{\mathrm{HO}}}$. The last relations can be rewritten as follows:

$$\dim R(J_{\mathrm{HO}}) + \dim N(J_{\mathrm{HO}}^{\mathrm{T}}) = \dim R(J_{\mathrm{AHO}}) + \dim N(J_{\mathrm{AHO}}^{\mathrm{T}}), \tag{5.1}$$

$$\dim R(J_{\mathrm{HO}}^{\mathrm{T}}) + \dim N(J_{\mathrm{HO}}) < \dim R(J_{\mathrm{AHO}}^{\mathrm{T}}) + \dim N(J_{\mathrm{AHO}}). \tag{5.2}$$

The rank of the transposed matrix is equal to the rank of the original matrix; therefore, with allowance for (5.1), inequality (5.2) can be written as follows:

$$\dim N(J_{\mathrm{HO}}) - \dim N(J_{\mathrm{HO}}^{\mathrm{T}}) < \dim N(J_{\mathrm{AHO}}) - \dim N(J_{\mathrm{AHO}}^{\mathrm{T}}). \tag{5.3}$$

Relation (5.3) can be rewritten as follows:

$$\dim N(J_{\mathrm{AHO}}^{\mathrm{T}}) < \dim N(J_{\mathrm{AHO}}) - \dim N(J_{\mathrm{HO}}) + \dim N(J_{\mathrm{HO}}^{\mathrm{T}}). \tag{5.4}$$

An important special case from the last expression follows. We have seen that in order to exclude the defectiveness of the grasp, it is necessary to ensure the strict equality $\dim N(J_{\mathrm{AHO}}^{\mathrm{T}}) = 0$. From (5.4) it follows that this equality is possible only if

$$\dim N(J_{\mathrm{AHO}}) - \dim N(J_{\mathrm{HO}}) + \dim N(J_{\mathrm{HO}}^{\mathrm{T}}) = 1. \tag{5.5}$$

Relation (5.5) can be used when analyzing the properties of the AHO manipulation system for a specified manipulator's configuration for a known (previously chosen) object's grasp by the HO system. In addition, relation (5.5) points on the principle possibility of choosing the manipulator's configuration such that $\dim N(J_{\mathrm{HO}}^{\mathrm{T}}) \neq 0$ but $\dim N(J_{\mathrm{AHO}}^{\mathrm{T}}) = 0$ and, therefore, in the AHO system, the grasp's defectiveness is excluded (it arises when planning a grasp in the HO system).

6. ENSURING THE DESIRED GRASP PROPERTIES IN THE AHO SYSTEM

Assume that a grasp is sufficient (i.e., $\dim N(G) \neq 0$) and certain (i.e., $\dim N(G^{\mathrm{T}}) = 0$). These conditions are the same for the HO and AHO systems.

Jacobians are included in relations that determine the conditions for certain desired properties of a grasp such as the exclusion of the defectiveness, along with the controllability of the movement (the forces acting on an object) and internal forces (forces of compression). These relations for the AHO system are presented as follows:

$$\dim N(J_{\mathrm{AHO}}^{\mathrm{T}}) = 0, \quad \operatorname{rank} GJ_{\mathrm{AHO}} = \operatorname{rank} G = n_v,$$
$$N(G) \cap N(J_{\mathrm{AHO}}^{\mathrm{T}}) = 0.$$

Taking into account the form of the matrix J_{AHO} (expression (4.1)), we specify the properties that the Jacobian J_{a} must have in order to exclude the defectiveness of the grasp in the AHO system and ensure the controllability of the movements of the hand's fingers, along with the controllability of the forces applied by the hand to the object.

1. The condition $\dim N(J_{\mathrm{AHO}}^{\mathrm{T}}) = 0$. The null-space of the matrix $J_{\mathrm{AHO}}^{\mathrm{T}}$ is formed by the vectors of the internal effects of the hand on an object λ_{e} that do not depend on the forces and moments acting on the joints of the hand fingers and the arm. Denote these effects by the block vector λ_{e} that satisfies the condition

$$J_{\mathrm{AHO}}^{\mathrm{T}} \lambda_{\mathrm{e}} = \mathbf{0} \quad \text{for} \quad \lambda_{\mathrm{e}} \neq \mathbf{0}. \tag{6.1}$$

Condition (6.1) can be rewritten as the simultaneous fulfilment of the equalities

$$J_{\mathrm{HO}}^{\mathrm{T}}\lambda_{\mathrm{e}} = \mathbf{0}, \quad J_{\mathrm{a}}^{\mathrm{T}}\lambda_{\mathrm{e}} = \mathbf{0} \quad \text{for} \quad \lambda_{\mathrm{e}} \neq \mathbf{0}.$$

It can be seen that if the last equalities are not fulfilled at a time with the identical (for them) nonzero value of the vector λ_{e} (i.e., $J_{\mathrm{HO}}^{\mathrm{T}}\lambda_{\mathrm{e}} = \vec{0}$, but $J_{\mathrm{a}}^{\mathrm{T}}\lambda_{\mathrm{e}} \neq \mathbf{0}$, and vice versa), then dim $N(J_{\mathrm{HO}}^{\mathrm{T}}) = 0$ and a grasp that is defective in the HO system, will not be defective in the AHO system. In other words, it is necessary that

$$N(J_{\mathrm{HO}}^{\mathrm{T}}) \cap N(J_{\mathrm{a}}^{\mathrm{T}}) = 0. \tag{6.2}$$

2. The fulfillment of the condition

$$\mathrm{rank}\, GJ_{\mathrm{AHO}} = \mathrm{rank}\, G = n_v$$

is indicative of the controllability both of the movements of the hand's fingers and of the force effects of the hand on an object in the AHO system.

Taking into account the form of the matrices G and J_{AHO} presented in expressions (2.5) and (4.1), we obtain

$$J_{\mathrm{AHO}}^{\mathrm{T}}G^{\mathrm{T}} = \begin{bmatrix} G_{11}J_{\mathrm{HO}1} & G_{12}J_{\mathrm{HO}2} & \ldots & G_{1m}J_{\mathrm{HO}m} & G_{11}J_{\mathrm{a}1} + G_{12}J_{\mathrm{a}2} + \ldots + G_{1m}J_{\mathrm{a}m} \\ G_{21}J_{\mathrm{HO}1} & G_{22}J_{\mathrm{HO}2} & \ldots & G_{2m}J_{\mathrm{HO}m} & G_{21}J_{\mathrm{a}1} + G_{22}J_{\mathrm{a}2} + \ldots + G_{2m}J_{\mathrm{a}m} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ G_{61}J_{\mathrm{HO}1} & G_{62}J_{\mathrm{HO}2} & \ldots & G_{6m}J_{\mathrm{HO}m} & G_{61}J_{\mathrm{a}1} + G_{62}J_{\mathrm{a}2} + \ldots + G_{6m}J_{\mathrm{a}m} \end{bmatrix}^{\mathrm{T}}.$$

Denote by $J_{\mathrm{a}}^{\mathrm{T}}G^{\mathrm{T}}$ the last row of the matrix $J_{\mathrm{AHO}}^{\mathrm{T}}G$. The null space of the matrix $J_{\mathrm{AHO}}^{\mathrm{T}}G^{\mathrm{T}}$ can be presented as a set of some nonzero vectors φ, each of which satisfies the condition

$$J_{\mathrm{AHO}}^{\mathrm{T}}G^{\mathrm{T}}\phi = \mathbf{0} \quad \text{for} \quad \phi \neq \mathbf{0}.$$

Given the structure of the matrix J_{AHO} (4.1), this condition can be presented as the joint fulfillment of the equalities

$$J_{\mathrm{HO}}^{\mathrm{T}}G^{\mathrm{T}}\phi = \mathbf{0}, \quad J_{\mathrm{a}}^{\mathrm{T}}G^{\mathrm{T}}\phi = \mathbf{0} \quad \text{for} \quad \phi \neq \mathbf{0}.$$

If the last equalities are not fulfilled at a time for the common (for both relations) value of $\phi \neq \mathbf{0}$ (i.e., $J_{\mathrm{HO}}^{\mathrm{T}}G^{\mathrm{T}}\phi = \mathbf{0}$, but $J_{\mathrm{a}}^{\mathrm{T}}G^{\mathrm{T}}\phi \neq \mathbf{0}$, and vice versa), then a grasp, which is uncontrollable in the HO system, can be controllable in the AHO system.

Thus, the controllability of a grasp in the AHO system requires the absence of values of $\phi \neq \mathbf{0}$ that are common for the null spaces of the matrices $J_{\mathrm{HO}}^{\mathrm{T}}G^{\mathrm{T}}$ and $J_{\mathrm{a}}^{\mathrm{T}}G^{\mathrm{T}}$:

$$N(J_{\mathrm{HO}}^{\mathrm{T}}G^{\mathrm{T}}) \cap N(J_{\mathrm{a}}^{\mathrm{T}}G^{\mathrm{T}}) = 0. \tag{6.3}$$

After planning the grasp is followed by the operation that consists in planning the trajectories of the joints of the hand and the arm and their movement along the planned trajectories. The movement is provided by control systems of the motion of the arm and the hand.

Example. As an example, consider a grasp of a flat object by a two-finger hand. An object in the shape of a quadrangle is grasped by the fingers on the side of the left and right faces (Fig. 2a). The left face is beveled at an angle of 45°. The dashed line shows the segment connecting the contact points, while the arrows show the direction of movement of the fingers. For illustration purposes, we assume that the object can perform only translational displacements in the direction of the *X*- and *Y*-axes of the *OXY* coordinate system associated with the stationary base. Here, $n_v = 2$. In Fig. 2a, q_1 and q_2 are the coordinates of the joints of the hand's fingers. In Fig. 2b, μ_1 and μ_2 are the forces of the actuators of the hand's fingers, λ_1 and λ_2 are the vectors of the forces applied to the object by the fingers at the grasping points, and P is the gravity vector of the object.

The vectors of forces acting on an object by the fingers are assigned in the frames associated with the object. The origins of these frames are located at the grasping points. Denote them by left frame and right frame. The *X*-axes of these frames are directed along normals to the left and right faces of the object in the direction to the object. The *Y*-axes are directed along the faces: up (left frame) and down (right frame).

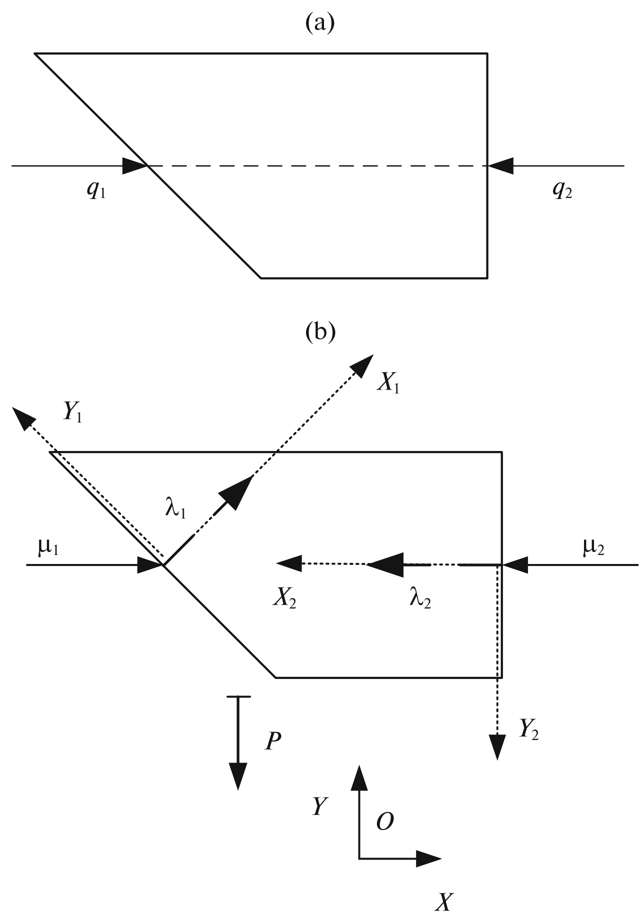

Fig. 2. Grasped flat object.

The contact on the right is a frictionless point contact. The contact on the left is also a point contact but with friction. In this case, the vectors λ_2 and v_{s2} have one nonzero component each, while the vectors λ_1 and v_{s1} have two nonzero components each; i.e., for the projections of these vectors on the X_1- and Y_1-axes we have

$$\lambda_1 = [\lambda_{1x}\,\lambda_{1y}]^{\mathrm{T}}, \quad v_{s1} = [v_{s1x}\,v_{s1y}]^{\mathrm{T}}, \quad \lambda_2 = [\lambda_{2x}\;0]^{\mathrm{T}}, \quad v_{s2} = [v_{s2x}\;0]^{\mathrm{T}}.$$

The vector v of the linear speeds of the center of mass of the object is specified in the frame of the base:

$$v = [v_x v_y]^{\mathrm{T}}.$$

In the analyzed example, the matrices G^{T} and J have the form

$$G^{\mathrm{T}} = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \\ -1 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & 0 \\ 0 & 1 \end{pmatrix}.$$

The dimensions of the rank and null spaces of the matrices G, G^{T}, J, J^{T}, and GJ are

$$\operatorname{rank}G = \operatorname{rank}G^{\mathrm{T}} = 2, \quad \dim N(G^{\mathrm{T}}) = 0,$$

$$\operatorname{rank}J = \operatorname{rank}J^{\mathrm{T}} = 2, \quad \dim N(J) = 0,$$

$$\operatorname{rank}GJ = 1, \quad \operatorname{rank}GJ \neq \operatorname{rank}G.$$

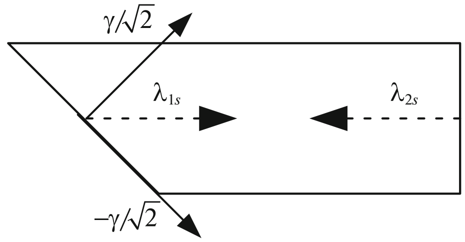

Fig. 3. Forces compressing an object.

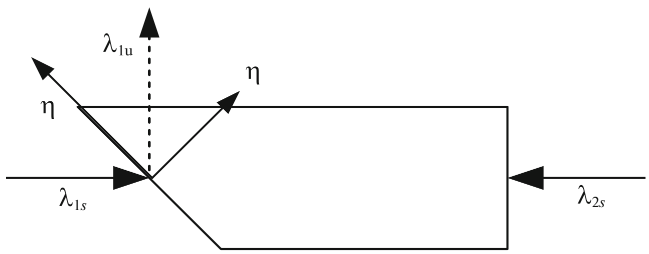

Fig. 4. Vector of uncontrollable force.

The null spaces of the matrices G, J^{T}, and GJ consist of the vectors

$$N(G) = [1/\sqrt{2}\ \ -1/\sqrt{2}\ \ 1]^{\mathrm{T}},\quad N(J^{\mathrm{T}}) = [1\ \ 1\ \ 0]^{\mathrm{T}},\quad N(GJ) = [1\ \ 1]^{\mathrm{T}}.$$

The analysis shows the following results:

(a) The grasp is not uncertain (the condition dim $N(G^{\mathrm{T}}) = 0$ is met); this corresponds to the absence of displacements of the object that are not associated with the movement of the fingers of the hand.

(b) The grasp is sufficient (the condition dim $N(G) \neq 0$); the null space of the matrix G forms the vector

$$N(G)= [1/\sqrt{2}\ \ -1/\sqrt{2}\ \ 1]^{\mathrm{T}}.$$

Its components are the forces $\lambda_{1s} = \gamma\,[1/\sqrt{2} - 1/\sqrt{2}]^{\mathrm{T}}$ and $\lambda_{2s} = \gamma[1\ 0]^{\mathrm{T}}$, which compress the object (see Fig. 3); γ is a multiplier.

(c) The grasp is defective, because dim $N(J^{\mathrm{T}}) \neq 0$, $N(J^{\mathrm{T}}) = [1\ 1\ 0]^{\mathrm{T}}$; this indicates the existence of an uncontrollable contact force: the vector $\lambda_{1\mathrm{u}} = \eta\,[1, 1]^{\mathrm{T}}$ (in Fig. 4, it is shown by the dotted line, while the components of the vector are shown using the thin lines), where η is a multiplier.

(d) The grasp is not redundant, because dim $N(J) = 0$.

(e) The movement of the object is not controllable, because rank $GJ \neq$ rank G.

Thus, by the criteria listed above, the grasp of an object by the HO system is not controllable and, at the same time, is defective.

Assume that the hand is attached to the manipulator. Consider for it a mechanism with one prismatic kinematic pair that moves the hand with a grasped object in the direction of the Y-axis of the basic frame.

The matrix J_{AHO} of such a system has the form

$$J = \begin{pmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \end{pmatrix}.$$

For the matrix GJ we obtain

$$GJ = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

We analyze the properties of the AHO system.

1. We can see that dim $N(J_{\rm AHO}) = \dim N(J^{\rm T}_{\rm AHO}) = 0$; consequently, the grasp in the AHO system is not redundant (as in the case of a grasp in the HO system). At the same time, the grasp in the AHO system is not defective (contrary to the grasp in the HO system). Due to this, there is a possibility to control all the contact forces.

2. The rank of the matrix GJ in the AHO system is rank $GJ_{\rm AHO}$ = rank $G = 2 = n_v$. This suggests that the ability to control the movements of an object is provided as follows: along the X-axis of the basic frame, due to the movement of the hand's fingers, and along the Y-axis, by moving the manipulator. The possibility to move an object along the Y-axis was not available in the HO system.

3. The condition $N(G) \cap N(J^{\rm T}) = 0$ is met, because dim $N(J)^{\rm T} = 0$. Hence, the AHO system provides the control of internal forces. Such a possibility was not available in the HO system.

The conditions presented in Sections 5 and 6 and required to ensure the desired properties are also met. In fact, since

$$\dim N(J^{\rm T}_{\rm HO}) - \dim N(J_{\rm HO}) = 1,$$

from (5.5) it follows that the condition dim $N(J_{\rm AHO}) = 0$ must be met. This condition is true.

Condition (6.2), namely $N(J^{\rm T}_{\rm HO}) \cap N(J^{\rm T}_{\rm a}) = 0$, is also met, since

$$N(J^{\rm T}_{\rm HO}) = [1\ \ 1\ \ 0]^{\rm T}, \quad N(J^{\rm T}_{\rm a}) = [1\ \ {-1}\ \ 0]^{\rm T}.$$

Condition (6.3) $N(J^{\rm T}_{\rm HO}G^{\rm T}) \cap N(J^{\rm T}_{\rm a}G^{\rm T}) = 0$ (which is equivalent to $N(GJ_{\rm HO})^{\rm T} \cap N(GJ_{\rm a})^{\rm T} = 0$) is also met, since

$$N(GJ_{\rm HO})^{\rm T} = [0\ \ 1]^{\rm T}, \quad N(GJ_{\rm a})^{\rm T} = N[0\ \ 1] = [0\ \ 1]^{\rm T}.$$

The example confirms the correctness of the relations obtained previously in the analytical form. The provision of the desired grasp properties is achieved by the movement of an object grasped by the hand, in the vertical direction. This movement is produced by means of the arm.

CONCLUSIONS

The theory of manipulation systems devotes considerable attention to the questions of object grasping by a gripper in the form of a multifinger hand. For systems of such a type, the relations [2] are found, which allow to estimate the grasp properties and quality measures. The foundation of these relations represent the grasp matrix G and the hand Jacobian. The first one makes it possible to determine whether we can move the grasped item relative to the hand's fingers, whereas J defines the hand's ability to hold an item under the action of external forces.

Grasp planning is usually carried out in the HO system. This paper proposes the approach of grasp planning when considering the AHO manipulation system. The relations for calculating the grasp and Jacobian matrices for the AHO systems are presented and the relation between these matrices and analogous matrices of the HO system is revealed. It is shown that when planning a grasp in the AHO system, the desired properties can be provided even in the case when these properties are not provided in the HO system. Formally, this is achieved by the proper choice of components of the arm Jacobian; and in practice, by the choice of the corresponding configuration of its kinematic chain.

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